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Queueing Modeling of a SAIS Model with Alert, Infection and Vaccination

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Abstract—*In this investigation, a SAIS (Susceptible-Alert-Infected-Susceptible) M/M/1 queueing model with alert, infection, and vaccination has been considered. The main focus of this work is on the vaccination to prevent the infection in a population. The vaccination is the power tool for the prevention of the diseases spreading over the population size. Susceptible person become alert when symptoms of the disease can be seen on them and the alert may be injected due to more infection. The provision of vaccination is provided in alert stage. Also alert may be change into infected person and infected may be again change into susceptible. The transition rates as followed by exponential distribution. A Markov model is developed by using inflow and outflow transition rates of the model. The transient state probabilities are evaluated by solving the transient state equations by using runge kutta method and which are further used for calculating the model performances. A numerical illustration is also provided to validate the model.*

Keywords*: SAIS Epidemic Model, Infection, Vaccination, Markov model, Runge kutta method.*

1. INTRODUCTION

Epidemic models play a critical role in disease dynamics for providing the internal assessment to current biological scenario. The provision of vaccination is proved helpful in the prevention of many epidemics with the recovering from the symptoms of these diseases. Markov modeling is very much useful for predicting the behavior of epidemic and disease models. Some remarkable work has been done by the research scholars in the field of epidemiology by considering markov and stochastic modeling. Longini *et al.* [1] did the statistical analysis of the stages of HIV infection using a Markov modeling. Gentleman *et al.* [2] considered a multi-state Markov models for analyzing incomplete disease history data with illustrations for HIV disease. Debanne *et al.* [3] developed a multivariate Markov chain model to project tuberculosis (TB) progression among different races in the country.

Trapman and Bootsma [4] established a relation between the spread of infectious diseases and the dynamics of M/G/1 queues with processor sharing. Bature *et al.* [5] described a Markov chain model to use for tracking the movement of the virus from one generation to another in a period of 20 years. Sweeting *et al.* [6] considered multistate Markov modelling to explore the rate at which the Hepatitis C disease progresses by taking the experimental data. Lee *et al.* [7] used a Markov chain analysis to model the progression of the disease among vulnerable people, infective people and AIDS cases for the two races separately. Oyewole [8] discussed a discrete-time Markov process for HIV/AIDs epidemic modeling is to determine the behavior of the epidemic and to keep it under control. Bortolussi [9] investigated models of contiuos time markov chains in which some populations are approximated continuously while others are considered ad discrete. Giamberardino and Iacoviello [10] considered SIR epidemic model with the dynamics of Susceptible, Infected & Removed subjects and an optimal vaccination strategy.

In the proposed work, an epidemic M/M/1 queueing model with alert, infection, vaccination and death has been taken into account. The provision of vaccination is applied to alert. The remaining paper is structured as follows. In section 2, we describe the model by explaining the assumptions, notations and the various states of the system. Section 3 presents the governing equations of the model by using the inflow and outflow state transition rates. The various system performances in terms of state probabilities are given in section 4. In section 5, a numerical illustration is facilitated to explore the validation of the model. Finally, conclusions are drawn in section 6.

2. MODEL DESCRIPTION

In this present work, we consider a epidemic queueing model with different states namely empty state, healthy state, alert state, vaccination state, infected state. We have consider a polpulation of size N which can be affected by an epiemic. Firstly, when the signs of this epidemic can be seeing in a healthy person, this stage at which he is not completely infected is known as alert. An alert person may be change into completely infected person by infected with this epidemic. The provision of the vaccination is also taken as a preventive tool to control on this epidemic. The vaccination is used when the healty person is affected partialy from the epidemic mean at alert state. All the transition rates from one state to another

state are taken as exponentially distributed. For the constraction of the model, the following assumptions are taken into consideration:

- The population size is taken as N i.e. there are total N living being in the system.
- Initially, the system is in empty state where there is no birth and no death and from this state after a birth system goes in the healty state.
- The birth and death of the persons are exponentially distributed with the rates λ and μ.
- The rate by which the healthy becomes alret is also exponentially distributed with the rate α_1 and the rate by which the infection take place is also exponentially distributed with the rate α_2.
- The rate by which the alert changes into infected is also follow the exponential distribution with the mean β.
- The vaccination is provided to alert persons with the exponentially distributed rates θ.
- After the success of the vaccination the alert persons become healty and infected persons may be become healthy with the exponentially distributed rates γ_1 and γ_2, respectively.

We develop the mathematical model by using some notations which are defined as below:

N	:	The population size.
Λ	:	The birth rate.
M	:	The death rate.
A	:	The partial infection rate (alert rate).
B	:	The transition rate from alert state to infected state.
Θ	:	The vaccination rate from the alert state.
γ_1	:	The transition rate from alert state to healthy state.
γ_2	:	The transition rate from infected state to healthy state.

Let $P_j(t)$ be the transient state probability that the system being in j^{th} $\left[j = 0,(H,i),(A,i),(V,i),(I,i), \text{where} i = 1,2,3,.....,N\right]$ state as shown in fig. 1 and j denotes the state of the system as follows:

0	The empty state at which there in no birth and no death takes place.
H, i	The healthy state where there are i (i=1, 2, 3,…., N) healthy persons in system.
A, i	The alert state where there are i (i=1, 2, 3,…., N) alert persons in system.
V, i	The vaccination state for alerts where there are i (i=1, 2, 3,…., N) alert persons in system.
I, i	The infection state where there are i (i=1, 2, 3,…., N) infected persons in system.

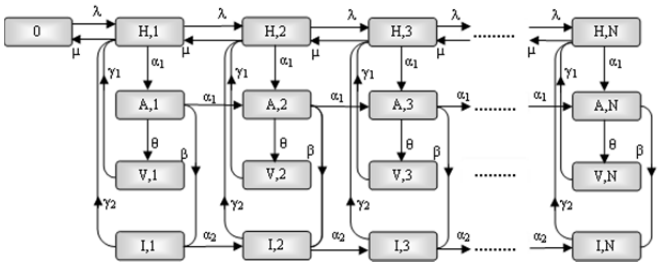

Fig. 1: State Transition Diagram

3. THE GOVERNING EQUATIONS

The steady state equations governing the epidemic model are constructed by equating the in-flow and out-flow (see fig. 1) as follows:

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu P_{H,1}(t) \quad (1)$$

$$\frac{dP_{H,1}(t)}{dt} = -(\lambda + \alpha_1 + \mu)P_{H,1}(t) + \lambda P_0(t) + \gamma_1 P_{V,1}(t) + \gamma_2 P_{I,1}(t) + \mu P_{H,2}(t) \quad (2)$$

$$\frac{dP_{H,i}(t)}{dt} = -(\lambda + \alpha_1 + \mu)P_{H,i}(t) + \lambda P_{H,i-1}(t) + \gamma_1 P_{V,i}(t) + \gamma_2 P_{I,i}(t) + \mu P_{H,i+1}(t)$$

, i=2, 3, …., N-1 (3)

$$\frac{dP_{H,N}(t)}{dt} = -(\alpha_1 + \mu)P_{H,N}(t) + \lambda P_{H,N-1}(t) + \gamma_1 P_{V,N}(t) + \gamma_2 P_{I,N}(t) \quad (4)$$

$$\frac{dP_{A,1}(t)}{dt} = -(\alpha_1 + \theta + \beta)P_{A,1}(t) + \alpha_1 P_{H,1}(t) \quad (5)$$

$$\frac{dP_{A,i}(t)}{dt} = -(\alpha_1 + \theta + \beta)P_{A,i}(t) + \alpha_1 P_{H,i}(t) + \alpha_1 P_{A,i-1}(t),$$

i =2, 3, ……., N-1 (6)

$$\frac{dP_{A,N}(t)}{dt} = -(\theta + \beta)P_{A,N}(t) + \alpha_1 P_{H,N}(t) + \alpha_1 P_{A,N-1}(t) \quad (7)$$

$$\frac{dP_{V,i}(t)}{dt} = -\gamma_1 P_{V,i}(t) + \theta P_{A,i}(t), \quad i=1, 2, 3, ……., N \quad (8)$$

$$\frac{dP_{I,1}(t)}{dt} = -(\alpha_2 + \gamma_2)P_{I,1}(t) + \beta P_{A,1}(t) \quad (9)$$

$$\frac{dP_{I,i}(t)}{dt} = -(\alpha_2 + \gamma_2)P_{I,i}(t) + \beta P_{A,i}(t) + \alpha_2 P_{I,i-1}(t),$$

i=2, 3, ……., N-1 (10)

$$\frac{dP_{I,N}(t)}{dt} = -\gamma_2 P_{I,N}(t) + \beta P_{A,N}(t) + \alpha_2 P_{I,N-1}(t) \quad (11)$$

The transient probabilities of different states have been obtained by solving equations (1)-(11) by fourth order Runge-Kutta technique.

4. PERFORMANCE MEASURES

In section, our main objective of our investigation is to predict various performance metrics in terms of the steady state probabilities which are obtained by using fourth order Runge-Kutta method in previous section. Some indices to characterize the system performance are as follows:

- The expected number of healthy persons in the epidemic system is

$$E(H) = \sum_{i=1}^{N} iP_{H,i}(t) \quad (12)$$

- The expected number of alert persons in the epidemic system is

$$E(A) = \sum_{i=1}^{N} iP_{A,i}(t) \quad (13)$$

- The expected number of infected persons in the epidemic system is

$$E(I) = \sum_{i=1}^{N} iP_{I,i}(t) \quad (14)$$

- The expected number of persons under the vaccination is

$$E(V) = \sum_{i=1}^{N} iP_{V_1,i}(t) \quad (15)$$

5. NUMERICAL ILLUSTRATION

For the validation of the model discussed in previous sections, we perform a numerical illustration for the transient analysis of the epidemic M/M/1 queueing model. We use Runge-Kutta technique (RKT) of fourth order for solving these equations, which is implemented by exploiting MATLAB's 'ode45' function. For this, we have considered a time span with equal intervals. The numerical results by varying different parameters for various performance measures are summarized in the graphical presentations are also provided. For the computation purpose, we fix the values of the some parameters as follows: λ=0.1, μ=0.2, α_1=0.0004, α_2=0.0005, β=0.3, γ_1=0.0002, γ_2=0.0001 for figures 2-4.

In graph 2, the effect of vaccination rate θ on the expected number of infected persons E(I) has been shown by varying time. It is realized that the expected number of infected persons E(I) is decreases as the vaccination rate θ increases with increasing time.

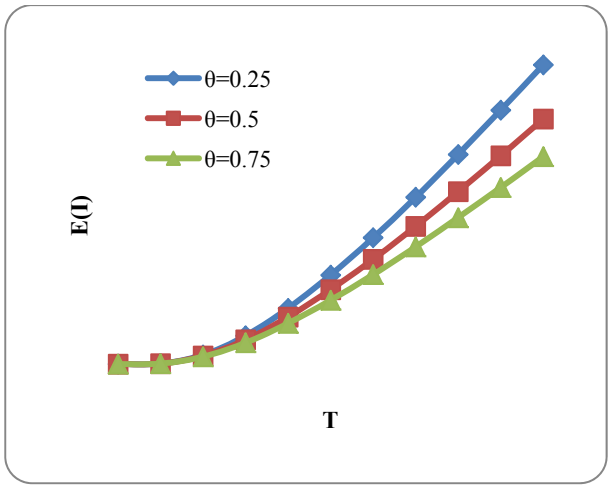

Fig. 2: Effect of θ on E(I)

In fig. 3, the effect of vaccination rate θ on the expected number of alert persons E(A) is shown with the increment of time. It is easily seen that E(A) is decreases as vaccination rate θ increases and the expected number of alert persons E(A) is increases as time increases .

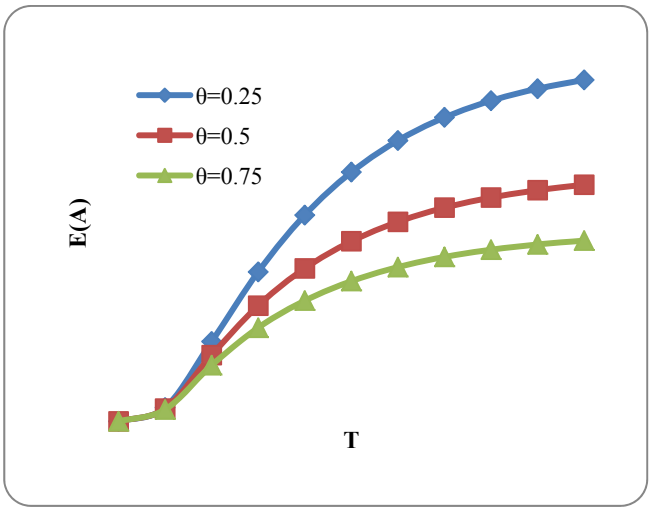

Fig. 3: Effect of θ on E(A)

In the last in fig. 4, the relationship between the vaccination rate θ and the expected number of persons under vaccination persons E(V) is drawn. It is found that the expected number of persons under vaccination E(V) is increases as the vaccination rate θ as well as time increases.

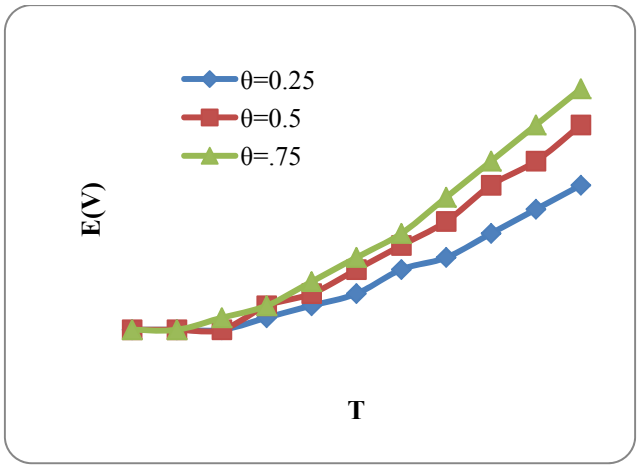

Fig. 4: Effect of θ on E(V)

6. CONCLUSION

In this paper, we deal with the modeling of an epidemic model with the use of M/M/1 queueing model. The various concepts such as alerts, infection and vaccination have been considered to cope with the real life issues. After constructing the model, the transient state equations are formed which are further used to find various system performances. We have also tried to provide a numerical example to find the numerical results for an epidemic model in the point of views of queueing theory. Also it is observed that the vaccination strategy improved the performance of the epidemic model. It is supposed that our investigation may be helpful for the analysis of epidemic model.

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